SY-27

ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2020

1/12

PART-I/II/III

SUBJECT: MATHEMATICS (SCIENCE)..........

CODE NO: ....

VERSION: ..

80 SCORES

2½ HOURS

Qn No	Sub Qns	Answer key/ value Points	Score	Total Score
1	(i)	(C) or (6,8) ∈ R	1	3
	(ii)	a * e = a a + e + 1 = a e + 1 = 0 e = -1	1 ½ ½	
2	(i)	A = $\begin{bmatrix} 4 & 0 \\ 8 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$ Remark: For any two non-zero matrix give mark	½ ½	
	(ii)	A' = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ A + A' = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ = $\begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$ $\frac{A + A'}{2} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 3 \end{bmatrix}$ A - A' = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ = $\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$ $\frac{A - A'}{2} = \begin{bmatrix} 0 & -3/2 \\ 3/2 & 0 \end{bmatrix}$ $\therefore A = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -3/2 \\ 3/2 & 0 \end{bmatrix}$ Remark: A= $\frac{1}{2}$(A + A') + $\frac{1}{2}$(A -A') give ½ mark Or A = P + Q form give ½ mark	½ ½ ½ ½	3

3		$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ $R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$ $= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2 - a^2 \\ 0 & c-a & c^2 - a^2 \end{vmatrix}$ = (b-a) (c – a) $\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$ =(b-a) (c-a) [c+a – (b+a)] =(b-a) (c-a) (c-b) =(a-b) (b-c) (c-a) Remark: For direct method give 1 mark	½ ½ ½ 1 ½	3
4	(i)	(b) or A continuous function is always differentiable	1	3
	(ii)	$x^2 + y^2 + xy = 100$ $2x + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$ $(2y + x)\frac{dy}{dx} = -(2x + y)$ $\frac{dy}{dx} = \frac{-(2x+y)}{2y + x}$	1½ ½	
5	(i)	(b) or |sinx|	1	3
	(ii)	Not differentiable. Because of not a smooth curve (or curve have sharp edges) Remark: (ii) For no also give mark.	1	
	(iii)	$Y = \sqrt{\tan x}$ $\frac{dy}{dx} = \frac{1}{2.\sqrt{\tan x}}.\sec^2 x$ Remark: $\frac{dy}{dx} = \frac{1}{2.\sqrt{\tan x}}$ give ½, $\frac{d\tan x}{dx} = \sec^2 x$ give ½	1	
6	(i)	(B) or 2 $y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$ $\therefore$ Slope of the tangent at $(0, 1)$ *is* $= 2e^0 = 2$	1	3
	(ii)	Equation of line in normal for is $y - y_1 = \frac{-1}{m}.(x - x_1)$	½	

		For y-3= $\frac{-1}{2}$(x-2) ⇒ **2y-6 = -x + 2** **X + 2y – 8 = 0** Alternative Method: Equation of tangent **y – 1 = 2 (x – 0)** **y – 1 = 2x** **y – 2x – 1 = 0** Equation of line ⊥ʳ to this line is **x + 2y + k = 0.** This passes through **(2, 3)** **2 + 2*3 + k = 0** **8 + k = 0** **k = - 8** ∴ Equation is $\mathbf{x + 2y - 8 = 0}$ Remark: Equation of the tangent -> ½ Equation of the line ⊥ʳ to this line -> 1/2	½ 1	
7	**(1)** **(ii)**	**(b) or 3** $\mathbf{y = e^{-3x}}$ $\mathbf{\frac{dy}{dx} = -3e^{-3x}}$ $\mathbf{\frac{d^2y}{dx^2} = 9e^{-3x}}$ $\mathbf{\frac{d^2y}{dx^2}} + \frac{dy}{dx} - 6y = \mathbf{9e^{-3x} + -3e^{-3x} - 6e^{-3x} = 0}$ ∴ $\mathbf{e^{-3x}}$ is a solution Remark: $\mathbf{\frac{dy}{dx} = e^{-3x}}$ -> ½	**1** **1** **½** **½**	**3**
8	**(i)** **(ii)**	**(b)** or **y=2** **z-4 = 0** **y – 2 + k (z – 4) = 0** This passes through **(2, 1, 2)** **1 – 2 + k (2 – 4) = 0** **-1 - 2k = 0** **2k = -1** **K = ½** ∴ **Equation is y – 2 –½ (2 – 4) = 0** **2y – 4 -z + 4 = 0** **2y – z = 0** Remark: Analyzing the problem give 1 mark	**1** **½** **½** **½** **½**	**3**

9	(i)	$f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$ $(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$ $x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$ $-3x_1 + 2x_2 = -2x_1 - 3x_2$ $-3x_1 + 2x_1 = -2x_2 - 3x_2$ $-x_1 = -x_2$ $x_1 = x_2$ $\therefore$ f is one to one Now $y = \frac{x-2}{x-3}$ $yx - 3y = x - 2$ $(y-1)x = 3y - 2$ $x = \frac{3y-2}{y-1} \in A$ $\therefore$ f is onto Remark: $f(x_1) = f(x_2)$ give ½ mark	½ ½ ½ ½	4
	(ii)	Yes. Because it is a bijection Remark: For yes also give 1 mark	1	
	(iii)	$f^{-1}(x) = \frac{3x-2}{x-1}$ Remark: $\frac{3y-2}{y-1}$ give ½ mark	1	
10	(i)	$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ or (b)	1	
	(ii)	$\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right) = \frac{\pi}{4}$ $\frac{5x}{1-6x^2} = \tan\frac{\pi}{4} = 1$ $5x = 1 - 6x^2$ $6x^2 + 5x - 1 = 0$ $x = \frac{1}{6}$ or $x = -1$	1 1 ½	

		X = -1 does not satisfy the equation as LHS of the equation becomes negative. So **x** = $\frac{1}{6}$ Remark: **tan** $\frac{\pi}{4} = 1$ **give ½ mark**	½	
11	**(i)**	$\mathbf{u = x^x}$ **and** $\mathbf{v = x^{sinx}}$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ **Now,** $\mathbf{u = x^x}$ log u = x log x $\frac{1}{u}\frac{du}{dx} = 1 + \log x$ $\frac{du}{dx} = u(1 + \log x) = x^x(1 + \log x)$ $\mathbf{v = x^{sinx}}$ **log v = sinx log x** $\frac{1}{v}\frac{dv}{dx} = \frac{\sin x}{x} + \log x.\cos x$ $\frac{dv}{dx} = v\left(\frac{\sin x}{x} + \log x.\cos x\right)$ $= x^{\sin x}\left(\frac{\sin x}{x} + \log x.\cos x\right)$ $\therefore \frac{dy}{dx} = x^x(1 + \log x) + x^{\sin x}\left(\frac{\sin x}{x} + \log x.\cos x\right)$	½ ½ ½ ½ ½ ½	4
	(ii)	y= xcosx. $\frac{dy}{dx} = -x\sin x + \cos x$ Remark: For product rule give ½ mark.If $\frac{dy}{dx}$ is correct, give 1 mark and it is not necessary to find the second derivative	1	
12	**(i)** **(ii)**	**(b)** or $\sec^2 x$ **f(x)=** $\int(4x^3 - \frac{3}{x^4})\,dx$ $= 4\frac{x^4}{4} - 3\cdot\frac{x^{-3}}{-3} + C$	1 1	4

		$= x^4 + \frac{1}{x^3} + C$	1	
		f(2) = 0	½	
		$0 = 2^4 + \frac{1}{2^3} + C$	½	
		$C = -\frac{129}{8}$		
		$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$		
		Remark: Integrating both the sides give ½ mark. f(x) with or without C with correct integration give 2 marks		
13	(i)	$\int_a^b ydx$ or (b)	1	
	(ii)	Area = $\int_0^{\frac{\pi}{4}} \sin x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx$	1	
		$= [-\cos x]_0^{\frac{\pi}{4}} + [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	1	4
		$= -[\cos \frac{\pi}{4} - \cos 0] + [\sin \frac{\pi}{2} - \sin \frac{\pi}{4}]$	½	
		$= -\left[\frac{1}{\sqrt{2}} - 1\right] + 1 - \frac{1}{\sqrt{2}}$		
		$= \frac{-1}{\sqrt{2}} + 1 + 1 \frac{-1}{\sqrt{2}} = 2 - \sqrt{2}$	½	
		Remark: Alternative methods: Area = 2. $\int_0^{\frac{\pi}{4}} \sin x dx = 2 - \sqrt{2}$		
14	(i)	y = mx	½	
		$\frac{dy}{dx} = m$	1	
		$\therefore y = x\frac{dy}{dx}$	½	
	(ii)	$P = \frac{1}{x}$ $Q = x^2$	1	4
		$IF = e^{\int pdx} = e^{\int \frac{1}{x}dx} = x$		
		Solution is		
		$y.IF = \int Q.IF\, dx + c$	½	
		$y.x = \int x^2\, xdx + c$	½	
		$= \frac{x^4}{4} + c$		
		Remark: Identifying linear equn give ½ mark.		

		IF = $e^{\int pdx}$ give ½ mark		
15		$\vec{AB} = i + 2j + 3k$ $\vec{AC} = 0i + 4j + 3k$ $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6i - 3j + 4k$ $\lvert \vec{AB} \times \vec{AC} \rvert = \sqrt{61}$ $\therefore \boldsymbol{c} = \frac{-6i - 3j + 4k}{\sqrt{61}}$ Remark: $\hat{n} = \frac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert}$ give ½ mark Alternative method: Equn of the plane in 3 point form give 3 marks $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-1 & y-1 & z-2 \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = 0$ => (x-1) (6-12) –(y-1) (3 -0) + (z-2) (4 - 0) =0 => 6x + 6 -3y +3 + 4z -8 = 0 => 6x + 3y -4z + 1 = 0 $\hat{n} = \frac{6i + 3j - 4k}{\sqrt{61}}$ give full mark	1 1 1 1	4
16	(i) (ii)	$\bar{r} = (-i - j - k) + \lambda(7i - 6j + k)$ $\bar{r} = (3i + 5j + 7k) + \mu(i - 2j + k)$ Remark: $\bar{r} = \bar{a} + \lambda \bar{b}$ give ½ mark $\bar{c} - \bar{a} = 3i + 5j + 7k + i + j + k$ **= 4i + 6j + 8k** $\bar{b} \times \bar{d} = \begin{vmatrix} i & j & k \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$ **= -4i -6j -8k** $\lvert \bar{b} \times \bar{d} \rvert = \sqrt{116}$ **S.D =** $\left\lvert \frac{(4i + 6j + 8k).(-4i - 6j - 8k)}{\sqrt{116}} \right\rvert$ $= \sqrt{116}$ Remark: Identifying $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ give 1 mark Alternative method: Cartesian method, correct answer give full mark Formula give ½ mark	1 ½ ½ ½ 1 ½	4
17	(i) (ii)	$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ $d = \left\lvert \frac{\frac{x_1}{a} + \frac{y_1}{b} + \frac{z_1}{c} - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \right\rvert$	1 1	4

	       (iii)	$= \dfrac{1}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}}}$ Remark: Distance formula give ½ mark $(\overline{r} - \overline{a}).\overline{\mathbf{N}} = \mathbf{0}$ $[\overline{r}$ – (i+0j-2k)**].**[i+j-k]=0 **[$(\mathrm{xi} + \mathrm{yj} + \mathrm{zk}) - (i + 0j - 2k)$].[i+j-k]=0** **x+y-z-3=0** **Remark: Alternative method: formula** $A(x-x_1)+ B(y-y_1) + C(z-z_1)$ **= 0 give ½ mark** **1(x-1) + 1(y-0) - 1(z+2) = 0 give ½ mark** x+y -z -3 =0 give ½ mark vector form give ½ mark	 1 1	
18	 **(i)** **(ii)** **(iii)** **(iv)**	**$P(A^1)$ = 0.7 $P(B^1)$ = 0.4 P(A) = 0.3 P(B) =0.6** **P(A ∩B) = P(A).P(B) = 0.3 x 0.6 = 0.18** **$P(A \cap B^1) = P(A).P(B^1)$ = 0.3 x 0.4 = 0.12** **P(AUB) = P(A)+P(B)-P(A).P(B) = 0.72** **$P(A^1 \cap B^1) = P(A^1).P(B^1)$ = 0.28** Remark: For formula give ½ mark P(A’), P(B’) give ½ mark For Alternative method and correct answer give full mark	 1 1 1 1	4
19	**(i)** **(ii)** **(iii)**	**A =** $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ **Remark: 2X3 general matrix give ½ mark.** **If one element is not correct give full mark** **A’ =** $\begin{vmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{vmatrix}$ **AA’ =** $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{vmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{vmatrix} = \begin{bmatrix} 29 & 38 \\ 38 & 50 \end{bmatrix}$ **(AA’)’ =** $\begin{bmatrix} 29 & 38 \\ 38 & 50 \end{bmatrix}$ **= AA’** **∴ AA’ is symmetric.** Remark: A = A’ give ½ mark For any matrix prove AA’ = (AA’)’ give 2 mark **(A+A’)’ = A’ + (A’)’** **= A’ + A** **= A + A’** **∴ A+A’ is symmetric.** Remark: Using any example give full mark	2 ½ 1 ½ 1 1	6

20	(i)	A' = - A $\|A'\| = \|-A\| = (-1)^3\ \|A\|$ $\|A'\| = -\ \|A\|$ $\|A\| = -\|A\|$ $\Rightarrow 2\|A\|=0$ $\|A\|=0$ Remark: For any skew symmetric and proving |A|=0 give 1 mark	1 ½ ½	6
	(ii)	$\begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & 5 \end{vmatrix} = 0$ $(2+x)(-5-2) - 3(5-2x) + 4(1+x) = 0$ $(2+x)\mathrm{x} - 7 - 3(5-2x) + 4(1+x) = 0$ $-14 - 7x - 15 + 6x + 4 + 4x = 0$ $3x - 25 = 0$ $x = \frac{25}{3}$ Remark: If |A|=0 is considered, then give ½ mark	½ ½ ½ ½	
	(iii)	$\|3AB\| = 3^2\|AB\|$ $= 9\ \|A\|.\|B\|$ = 9x-1x3 =-27 Remark: If |AB|=|A||B| then give ½ mark	½ ½ ½ ½	
21	(i)	$f(x) = x^2 + 2x - 5$ $f'(x) = 2x + 2$ $2x + 2 = 0$ $x = -1$ Interval is $(-\infty, -1)$, $(-1, \infty)$ $in\ (-1, \infty)$, $f'(x) > 0$ ∴ Strictly increasing $in\ (-\infty, -1)$ $f'(x) < 0$ ∴ Strictly decreasing	 1 ½ ½	6
	(ii)	$y = x^3$ $\frac{dy}{dx} = 3x^2$ $(\frac{dy}{dx})(1,1) = 3$ Equation of tangent $y - 1 = 3(x - 1)$ $y - 3x + 2 = 0$ Equation of normal $y - 1 = \frac{-1}{3}(x - 1)$ $3y + x - 4 = 0$	½ ½ ½ ½	

	(iii)	Remark: If m=$\frac{dy}{dx}$ then give ½ mark For formula give ½ mark **h'(x)= cosx – sinx** **h'(x) = 0 $\Rightarrow$ sinx = cosx $\Rightarrow$ x=$\frac{\pi}{4}$** **h'($\frac{\pi}{6}$) =** $\frac{\sqrt{3}-1}{2}$ >0 **h'($\frac{\pi}{3}$) =** $\frac{1-\sqrt{3}}{2}$ <0 Max: value = $\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \sqrt{2}$ $\therefore$ **h(x) has local max at x=$\frac{\pi}{4}$** Remark: For writing conditions of max: min: give 1 mark	½ ½ ½ ½	
22	(i) (ii) (iii)	$\int \frac{dx}{1+\frac{x^2}{4}}$ = $\int \frac{dx}{\frac{1}{4}(4+x^2)}$ =4$\int \frac{dx}{x^2+4}$ =4. $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$ =$2\tan^{-1}\frac{x}{2} + c$ Remark: Alternative method: $\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx = \frac{\tan^{-1}\left(\frac{x}{2}\right)}{\frac{1}{2}}$ For formula give ½ mark $\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$ ***x*** **= A(x-2) + B(x-1)** ***x*** **=2 $\Rightarrow$ 2=B** ***x*** **=1 $\Rightarrow$ 1= -A $\Rightarrow$ A = -1** $\int \frac{x}{(x-1)(x-2)}dx = \int \left(\frac{-1}{x-1} + \frac{2}{x-2}\right)dx$ =-log|x-1| + 2 log|x-2| + c Remark: For any correct method and correct answer give full mark For formula give ½ mark $\int_0^{\frac{\pi}{2}} x\cos x\, dx = [x.\sin x]_0^{\frac{\pi}{2}} - [-\cos x]_0^{\frac{\pi}{2}}$ = $(\frac{\pi}{2}\sin\frac{\pi}{2} - 0) + (\cos\frac{\pi}{2} - \cos 0)$ = $\frac{\pi}{2} - 1$ For formula give ½ mark	 ½ ½ ½ ½ ½ ½ ½ ½ 1 1	6

23

(i) **(b) or 0** — 1

(ii) $\bar{a}.\bar{b} = |\bar{a}|.|\bar{b}|\cos\theta$

$= \mathbf{2.3.}\cos\theta$ — 1

When $\theta = 0$, $\bar{a}.\bar{b} = 6$

Or (c)

(iii) $\bar{b}\ x\ \bar{c} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3i - 7j - k$ — 1

$[\bar{a}\quad \bar{b}\quad \bar{c}] = \bar{a}\,.(\bar{b} \times \bar{c})$

$= \bar{a}\ .\ (3i - 7j - k)$ — 1

$= \mathbf{1.}\ \sqrt{59}\cos\theta$ — 1

When $\theta = 0$, $[\bar{a}\quad \bar{b}\quad \bar{c}] = \sqrt{59}$ — 1

Remark: For any unit vector a and finding [a b c] give 3, and for correct answer full mark.

$[a\ b\ c] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ give 1 mark

Total: 6

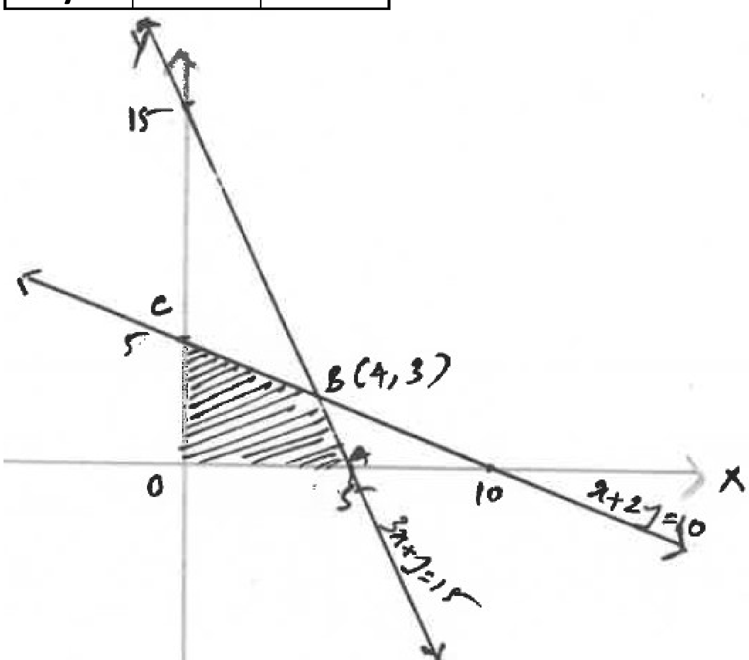
24

x + 2y = 10

x	0	10
y	5	0

3x + y = 15 — 1

x	0	5
y	15	0



3

Total: 6

B is (4, 3)

Vertices Z= 3x+2y

Vertices	Z = 3x+2y
O(0,0)	0
A(5,0)	15
B(4,3)	18
C(0,5)	10

1

Max: Z=18 at (4,3) X=4 y=3 **1**

Remark: For tabular column 1 mark, Figure, for each correct line 1 mark each, correct shading 1 mark.
For any 3 correct corner points give 1 mark.

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25 (i)

6k + 0.1 = 1 ½

6k = 0.9

$K = \frac{0.9}{6} = 0.15$ ½

Remark: Σpi=1, give ½ mark.

(ii)

$P(1<x<4) = P(2) + P(3)$ ½

$= 2k + 2k$

$= 4k$

$= 4 \times 0.15 = 0.6$ ½

(iii)

x	P(x)	xP(x)	$x^2P(x)$
0	0.10	0.00	0.00
1	0.15	0.15	0.15
2	0.30	0.60	1.20
3	0.30	0.90	2.70
4	0.15	0.60	2.40
		2.25	6.45

2

Mean = ΣxP(x) ½

= 2.25 ½

$V(x) = \Sigma x^2P(x) - (\Sigma xP(x))^2$ ½

$= 6.45 - (2.25)^2$ ½

$= 6.45 - 5.0625 = 1.3875$

Total: **6**

Summarized by: Sreemary K, (9446512086)